Name: Answer Key Date:

SEQUENCES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given each of the following sequences defined by formulas, determine and label the first four terms. A variety of different notations is used below for practice purposes.

(a) $f(n)=7n+2$

$a_1=7(1)+2=9$
$a_2=7(2)+2=16$
$a_3=7(3)+2=23$
$a_4=7(4)+2=30$

(b) $a_n=n^2-5$

$a_1=(1)^2-5=-4$
$a_2=(2)^2-5=-1$
$a_3=(3)^2-5=4$
$a_4=(4)^2-5=11$

(c) $t(n)=\left(\frac{2}{3}\right)^n$

$t_1=(2/3)^1=2/3$
$t_2=(2/3)^2=4/9$
$t_3=(2/3)^3=8/27$
$t_4=(2/3)^4=16/81$

(d) $t_n=\frac{1}{n+1}$

$t_1=1/(1+1)=1/2$
$t_2=1/(2+1)=1/3$
$t_3=1/(3+1)=1/4$
$t_4=1/(4+1)=1/5$

2. Sequences below are defined recursively. Determine and label the **next** three terms of the sequence.

(a) $f(1)=4$ and $f(n)=f(n-1)+8$

$f(2)=f(1)+8=4+8=12$
$f(3)=f(2)+8=12+8=20$
$f(4)=f(3)+8=20+8=28$

(b) $a(n)=a(n-1)\cdot\frac{1}{2}$ and $a(1)=24$

$a(2)=a(1)\cdot\frac{1}{2}=24\cdot\frac{1}{2}=12$
$a(3)=a(2)\cdot\frac{1}{2}=12\cdot\frac{1}{2}=6$
$a(4)=a(3)\cdot\frac{1}{2}=6\cdot\frac{1}{2}=3$

(c) $b_n=b_{n-1}+2n$ with $b_1=5$

$b_2=b_1+2(2)=5+4=9$
$b_3=b_2+2(3)=9+6=15$
$b_4=b_3+2(4)=15+8=23$

(d) $f(n)=2f(n-1)-n^2$ and $f(1)=4$

$f(2)=2f(1)-2^2=2(4)-4=8-4=4$
$f(3)=2f(2)-3^2=2(4)-9=8-9=-1$
$f(4)=2f(3)-4^2=2(-1)-16=-2-16=-18$

3. Given the sequence 7, 11, 15, 19, ..., which of the following represents a formula that will generate it?

(1) $a(n)=4n+7$ (3) $a(n)=3n+7$
(2) $a(n)=3n+4$ (4) $a(n)=4n+3$

Until we review arithmetic sequences, the easiest and safest way to do this problem is by guessing and checking. Substitute n = 1, 2, 3, … into each formula and see if it generates 7, 11, 15, 19, …

(4)

4. A recursive sequence is defined by $a_{n+1}=2a_n-a_{n-1}$ with $a_1=0$ and $a_2=1$. Which of the following represents the value of a_5?

(1) 8 (3) 3
(2) −7 (4) 4

$a_3=2a_2-a_1=2(1)-0=2-0=2$
$a_4=2a_3-a_2=2(2)-1=4-1=3$
$a_5=2a_4-a_3=2(3)-2=6-2=4$

(4)

5. Which of the following formulas would represent the sequence 10, 20, 40, 80, 160, …

(1) $a_n=10^n$ (3) $a_n=5(2)^n$
(2) $a_n=10(2)^n$ (4) $a_n=2n+10$

Although (2) might seem like the correct choice, if we substitute n = 1 into its equation, we get 20, not 10. In fact, choice (3) is correct as can be seen by substituting n = 1, 2, 3, …

(3)

$$\underset{a_1}{\underline{2}} \quad \underset{a_2}{\underline{5}} \quad \underset{a_3}{\underline{3}} \quad \underset{a_4}{\underline{-2}} \quad \underset{a_5}{\underline{-5}}$$

$n-2 \quad n-1 \quad n$

$$f(n) = f(n-1) - f(n-2)$$

5(1) 5(2) 5(3) 5(4)

6. For each of the following sequences, determine an algebraic formula, similar to *Exercise* #4, that defines the sequence.

3^1 3^2 3^3 3^4

(a) 5, 10, 15, 20, …

$5 = 5\cdot 1, 10 = 5\cdot 2, 15 = 5\cdot 3, \ldots$

$a(n) = 5n$

(b) 3, 9, 27, 81, …

$3 = 3^1, 9 = 3^2, 27 = 3^3, 81 = 3^4, \ldots$

$a(n) = 3^n$

(c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$\frac{1}{2} = \frac{1}{1+1}, \frac{2}{3} = \frac{2}{2+1}, \frac{3}{4} = \frac{3}{3+1}, \ldots$

$a(n) = \frac{n}{n+1}$

7. For each of the following sequences, state a recursive definition. Be sure to include a starting value or values.

$f(n) = f(n-1) - 2$ $f(n) = 3f(n-1)$

(a) 8, 6, 4, 2, …

Each term is two less than the previous:

$a_1 = 8$ and $a_{n+1} = a_n - 2$

(b) 2, 6, 18, 54, …

Each term is three times the previous:

$a_1 = 2$ and $a_{n+1} = 3a_n$

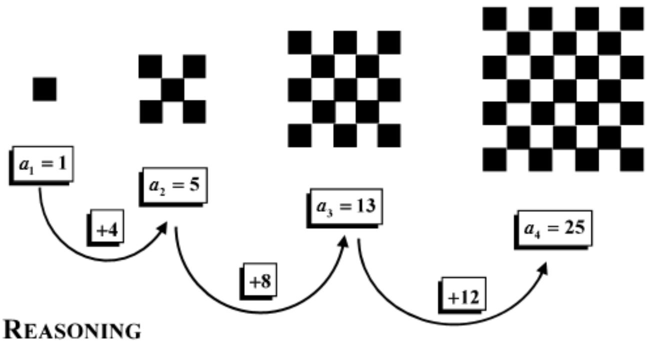
(c) $2, -2, 2, -2, \ldots$

Each term is the previous term multiplied by -1:

$a_1 = 2$ and $a_{n+1} = -1a_n$

APPLICATIONS

8. A tiling pattern is created from a single square and then expanded as shown. If the number of squares in each pattern defines a sequence, then determine the number of squares in the seventh pattern. Explain how you arrived at your choice. Can you write a recursive definition for the pattern?



To generate each term in this sequence, we are adding the next larger multiple of 4. So:

$a_5 = a_4 + 16 = 25 + 16 = 41$

$a_6 = a_5 + 20 = 41 + 20 = 61$

$a_7 = a_6 + 24 = 61 + 24 = 85$

A general recursive definition is:

$a_1 = 1$

$a_n = a_{n-1} + 4(n-1)$

REASONING

9. Consider a sequence defined similarly to the Fibonacci, but with a slight twist:

$$f(n) = f(n-1) - f(n-2) \text{ with } f(1) = 2 \text{ and } f(2) = 5$$

Generate terms $f(3), f(4), f(5), f(6), f(7)$, and $f(8)$. Then, determine the value of $f(25)$.

$f(3) = f(2) - f(1) = 5 - 2 = 3$
$f(4) = f(3) - f(2) = 3 - 5 = -2$
$f(5) = f(4) - f(3) = -2 - 3 = -5$
$f(6) = f(5) - f(4) = -5 - (-2) = -3$
$f(7) = f(6) - f(5) = -3 - (-5) = 2$
$f(8) = f(7) - f(6) = 2 - (-3) = 5$

Notice that the pattern starts to repeat itself in a cycle (period) of length 6. Thus, since 25 is 4 cycles of 6 past 1, $f(25)$ would have the same value as $f(1)$:

$f(25) = f(1) = 2$

1) Find the next two terms of each sequence. Then describe the pattern. The equations will be completed later.

1, 3, 5, 7, 9, 11, 13 Description: Add 2 ~~Equation: ____~~

2, 7, 12, 17, 22, 27, 32 Description: Add 5 ~~Equation: ____~~

-416, -323, -230, -137, -44, 49 Description: Add 93 ~~Equation: ____~~

-2, -5, -8, -11, -14, -17 Description: Add -3 ~~Equation: ____~~

2) So we take the **1st term**, and keep adding the same number over and over again. This is the definition of an **arithmetic sequence**.

3) **(the nth term) = (the 1st term) + (the common difference)(n – 1)**

4) In other words… $a_n = a_1 + d(n-1)$

d= common differemce
n= term number

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5) Examine the following sequence: 2, 7, 12, 17, 22, …

a] What is the **1st term**? 2

b] What is the **common difference**? 5

c] What is the 35^{th} term?

n

$a_{35} = 2 + 5(35-1) = \boxed{172}$

d] What is the formula to find the **nth** term?

$$\boxed{a_n = 2 + 5(n-1)}$$

$$a_n = 2 + 5n - 5$$

$$\boxed{a_n = -3 + 5n}$$ Alt.

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6) Examine the following sequence: 100, 90, 80, 70, 60, …

a] What is the **1st term**? 100

b] What is the **common difference**? −10

c] What is the 28th term?

$a_{28} = 100 + (-10)(28-1) = \boxed{-170}$

d] What is the formula to find the **nth** term?

$a_n = 100 - 10(n-1)$

***So the **common difference can be positive OR negative!!!**

7) How many terms are in the sequence: 7, 10, 13, … 55 ?

a_1 = 7, +3, a_n = 55

$55 = 7 + 3(n-1)$

-7

$\frac{48}{3} = \frac{3(n-1)}{3}$

$16 = n - 1$

$n = 17$

8) Insert 3 arithmetic means between 7 and 23.

Sum of an Arithmetic Series

Given an arithmetic series with n terms, $\{a_1, a_2, ..., a_n\}$, then its sum is given by:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$S_n = \frac{n(a_1+a_n)}{2}$

n = number of terms
a_1 = First term
a_n = Last term

9) Which of the following is the sum of the first 100 natural numbers? Show the process that leads to your choice.

(1) 5,000 (3) 10,000
(2) 5,100 (4) 5,050

$n = 100$
$a_1 = 1$
$a_n = 100$

$\text{Sum} = \frac{100}{2}(1+100)$

10) Find the sum of each arithmetic series described or shown below.

(a) The sum of the sixteen terms given by:

$$-10 + -6 + -2 + \cdots + 46 + 50.$$

$n = 16$
$a_1 = -10$
$a_{16} = 50$

$\text{sum} = \frac{16}{2}(-10+50)$ $\boxed{\text{sum} = 320}$

(b) The first term is -8, the common difference, d, is 6 and there are 20 terms

$n = 20$
$a_1 = -8$
$a_{20} = 106$

Find a_{20}
$a_{20} = -8 + 6(20-1)$
$a_{20} = 106$

$\text{Sum} = \frac{20}{2}(-8+106)$
$\boxed{\text{Sum} = 980}$

(c) The last term is $a_{12} = -29$ and the common difference, d, is -3.

$n = 12$
$a_1 = 4$
$a_{12} = -29$

Find a_1 $\rightarrow$ $a_n = a_{12} \rightarrow n = 12$

$-29 = a_1 + (-3)(12-1)$ $\rightarrow$ $-29 = a_1 - 33$
$+33 \quad +33$
$4 = a_1$

$\text{Sum} = \frac{12}{2}(4+(-29))$
$\boxed{\text{Sum} = -150}$

(d) The sum $5 + 8 + 11 + \cdots + 77$.

$+3$

$n = 25$
$a_1 = 5$
$a_n = 77$

Find n:
$77 = 5 + 3(n-1)$
$-5 \quad -5$
$72 = 3(n-1)$
$\frac{72}{3} = \frac{3(n-1)}{3}$
$24 = n - 1$
$25 = n$

$\text{Sum} = \frac{25}{2}(5+77)$
$= \boxed{1025}$

$$\begin{array}{rccccc} & 1 & 2 & 3 & 4 & 5 \\ + & 5 & 4 & 3 & 2 & 1 \\ \hline & 6 & 6 & 6 & 6 & 6 \end{array}$$

$$5 \times 6 = \frac{30}{2} = 15$$

$$a_n = a_1 + d(n-1)$$

$$a_1 \to -8 \qquad d \to 6 \qquad n \to 20$$

a_2

100, 110, 120, 130 . . . .

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11) Kirk has set up a college savings account for his son, Maxwell. If Kirk deposits $100 per month in an account, increasing the amount he deposits by $10 per month each month, then how much will be in the account after 10 years?

d

n = 120 (12 times/year for 10 years)

$a_1 = 100$

$a_n = 1290$

$a_n = 100 + 10(120 - 1)$

$a_n = 1290$

$\text{Sum} = \frac{120}{2}(100 + 1290)$

$\boxed{\text{Sum} = \$83{,}400}$

HW: Complete the homework

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Homework:

1. Which of the following represents the sum of $3+10+\cdots+87+94$ if the arithmetic series has 14 terms?

   (1) 1,358 (3) 679

   (2) 658 (4) 1,276

   $n = 14$

   $a_1 = 3$

   $a_{14} = 94$

   $= \frac{14}{2}(3+94)$

2. The sum of the first 50 natural numbers is

   (1) 1,275 (3) 1,250

   (2) 1,875 (4) 950

   $n = 50$

   $a_1 = 1$

   $a_{50} = 50$

   $\text{Sum} = \frac{50}{2}(1+50)$

3. If the first and last terms of an arithmetic series are 5 and 27, respectively, and the series has a sum 192, then the number of terms in the series is

   (1) 18 (3) 14

   (2) 11 (4) 12

   $192 = \frac{n}{2}(5+27)$

   $192 = \frac{n}{\cancel{2}}(\cancel{32})$

   $192 = 16n$

   $n = 12$

4. Find the sum of each arithmetic series described or shown below.

   (a) The sum of the first 100 even, natural numbers.

   (b) The sum of multiples of five from 10 to 75, inclusive.

   (c) A series whose first two terms are -12 and -8, respectively, and whose last term is 124.

   (d) A series of 20 terms whose last term is equal to 97 and whose common difference is five.

5. For an arithmetic series that sums to 1,485, it is known that the first term equals 6 and the last term equals 93. *Algebraically* determine the number of terms summed in this series.

APPLICATIONS

6. Arlington High School recently installed a new black-box theatre for local productions. They only had room for 14 rows of seats, where the number of seats in each row constitutes an arithmetic sequence starting with eight seats and increasing by two seats per row thereafter. How many seats are in the new black-box theatre? Show the calculations that lead to your answer.

7. Simeon starts a retirement account where he will place \$50 into the account on the first month and increasing his deposit by \$5 per month each month after. If he saves this way for the next 20 years, how much will the account contain in principal?

8. The distance an object falls per second while only under the influence of gravity forms an arithmetic sequence with it falling 16 feet in the first second, 48 feet in the second, 80 feet in the third, etcetera. What is the total distance an object will fall in 10 seconds? Show the work that leads to your answer.

9. A large grandfather clock strikes its bell once at 1:00, twice at 2:00, three times at 3:00, etcetera. What is the total number of times the bell will be struck in a day? Use an arithmetic series to help solve the problem and show how you arrived at your answer.

10) Examine the following sequence: 3, 7, 11, 15, 19, …

a] What is the **1st term**?

b] What is the **common difference**?

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c] What is the 13^{th} term?

d] What is the formula to find the **nth** term?

11) Examine the sequence: 111, 99, 87, 75, 63, …

a] What is the **1st term**?

b] What is the **common difference**?

c] What is the 40^{th} term?

d] What is the formula to find the **nth** term?

12) Find the 311^{th} term in the sequence: 7, 25, 43, 61, …

13) Find the 90^{th} term in the sequence: 100, 97, 94, 91, …

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